QUIZ #5 – Solutions
Each problem is worth 5 points

15 points total

1.

The constraint $x^2+y^2+z^2=9$ is a **closed** surface (a sphere). We define the Lagrangian

$$L(x,y,z,\lambda)=x^3+y^3+z^3+\lambda(x^2+y^2+z^2-9).$$

For critical points of L, we solve

$$0=\frac{\partial L}{\partial x}=3x^2+2\lambda x,\quad 0=\frac{\partial L}{\partial y}=3y^2+2\lambda y,\quad 0=\frac{\partial L}{\partial z}=3z^2+2\lambda z,\quad 0=\frac{\partial L}{\partial \lambda}=x^2+y^2+z^2-9.$$

Critical points (x,y,z) are $(\pm 3,0,0)$, $(0,\pm 3,0)$, $(0,0,\pm 3)$, $(0,\pm 3/\sqrt{2},\pm 3/\sqrt{2})$, $(\pm 3/\sqrt{2},0,\pm 3/\sqrt{2})$, $(\pm 3/\sqrt{2},\pm 3/\sqrt{2},0)$, $(\pm\sqrt{3},\pm\sqrt{3},\pm\sqrt{3})$. Since $f(x,y,z)=\pm 27$ at the first six critical points, $f(x,y,z)=\pm 27/\sqrt{2}$ at the second set of six critical points, and $f(\pm\sqrt{3},\pm\sqrt{3},\pm\sqrt{3})=\pm 9\sqrt{3}$, maximum and minimum values of $f(x,y,z)$ are ± 27.

2.

If we set $u=f(x,y,z,t)$, then

$$du=\frac{\partial f}{\partial x}dx+\frac{\partial f}{\partial y}dy+\frac{\partial f}{\partial z}dz+\frac{\partial f}{\partial t}dt=(y+t)\,dx+(x+z)\,dy+(y+t)\,dz+(z+x)\,dt.$$

3.

$$\begin{aligned}
\text{With}\quad f(0,1)&=0,\\
f_x(0,1)&=[2(x+y)\ln(x+y)+x+y]_{|(0,1)}=1,\\
f_y(0,1)&=[2(x+y)\ln(x+y)+x+y]_{|(0,1)}=1,\\
f_{xx}(0,1)&=[2\ln(x+y)+2+1]_{|(0,1)}=3,\\
f_{xy}(0,1)&=[2\ln(x+y)+2+1]_{|(0,1)}=3,\\
f_{yy}(0,1)&=[2\ln(x+y)+2+1]_{|(0,1)}=3,
\end{aligned}$$

$$\begin{aligned}
(x+y)^2\ln(x+y)&=x+(y-1)+\frac{1}{2!}[3x^2+6x(y-1)+3(y-1)^2]+\cdots\\
&=\frac{1}{2}[2x+2(y-1)+3x^2+6x(y-1)+3(y-1)^2]+\cdots\ .
\end{aligned}$$